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## MODELLING BALANCE OF THE EARTH ANGLE MOMENT, ATMOSPHERIC PROCESSES AND RADIOWAVEGUIDES: ADVANCED NON-STATIONARY THEORY

*There are presented the elements of an advanced non-stationary theory of the global mechanisms in the atmosphere low frequency processes, the Earth angle moment balance, teleconnection effects, and the radio-waveguides.*

**Keywords:** *microsystem technology, GeoMath, Earth angle moment balance, atmospheric models, teleconnection*

**Introduction.** In [1-3] (see also Ref. [4-14]) we presented the fundamental basis's of the new geophysical microsystem technology "GeoMath", in particular with the implementation of, new models of global mechanisms in atmospheric low-frequency processes, assessment of the balance of the angular momentum of the Earth and the effects telekonnectsii as well as the parameters of ultrashort radio waveguides. In this paper, the ongoing work [2,3], we will present the elements of a new advanced non-stationary theory of global mechanisms in atmospheric low-frequency processes, the balance of the angular momentum of the Earth, teleconnection effects and atmospheric radio waveguides. Let us remind that one of the key purposes (implemented into Microsystem Technology "GeoMath") focused on the discovery and testing of new predictors for long-term and very long-forecasts of low-frequency atmospheric processes. We are talking about the adaptation of the advanced theory of atmospheric macroturbulence applicable to atmospheric radiofrequency with a view to their possible using along with other as predictors in the long term. The preliminary PC experiments have demonstrated an effectiveness of a new advanced theory in application to modeling balance of angular momentum, the atmospheric moisture turnover in relation to the genesis of tropospheric radio waveguides and succession processes of atmospheric circulation forms (teleconnection, front-genesis) in order to develop new practical sensors in long-term forecasting and modeling of low-frequency atmospheric processes (see also Ref. [15-22]).

**Advanced non-stationary theory for balance of angular momentum.** An advanced non-stationary angular momentum balance equation of in the planetary dynamic movements of air masses is written in the following standard integral form [5,17]:

$$\begin{aligned} \frac{\partial}{\partial t} \int \rho M dV = & \int_{\varphi_1}^{\varphi_2} \int_0^H \int_0^{2\pi} \rho v M d\varphi dz d\lambda + \int_0^H \int_{\varphi_1}^{\varphi_2} \int_0^{2\pi} \left( p_E^i - p_W^i \right) a \cos \varphi dz d\varphi d\lambda + \\ & + \int_{\varphi_1}^{\varphi_2} \int_0^{2\pi} \int_0^H \tau_0 a \cos \varphi d\varphi d\lambda 2\pi, \end{aligned} \quad (1)$$

where  $M = \Omega a^2 \cos \varphi + u a \cos \varphi$  - angular momentum;  $\Omega$  - the angular velocity of rotation of the Earth;  $a$  - radius of the Earth;  $\varphi$  - Latitude ( $\varphi_1 - \varphi_2$  - separated latitudinal belt between the Arctic and polar fronts);  $\lambda$  - longitude;  $u, v$  - zonal and meridional components of the wind speed;  $\rho$  - air density;  $V$  - the entire volume of the atmosphere in this latitude belt from sea level to the average height of the elevated troposphere waveguide -  $H$  (in notations by A.Oort

$H = \infty$  [17]);  $p_E^i - p_W^i$  - the pressure difference between the eastern and western slopes of the  $i$ -th mountains;  $z$  - height above sea level;  $\tau_0$  - the shear stress on the surface.

From the point of view of physics, the cycle of balance of angular momentum in the contact zones with the hydrosphere and lithosphere becomes a singularity. This singularity can be detected through the occurrence of zones of fronts and soliton-type front. Then the kernel of equation (1) can be defined in the density functional ensemble of complex velocity potential [13]

$$w = \overline{v_\infty} z + \frac{1}{2\pi} \sum_{k=1}^n q_k \ln(z - a_k) + \frac{1}{2\pi} \sum_{k=1}^p \frac{M_k e^{\alpha_k i}}{z - c_k} - \frac{i}{2\pi} \sum_{k=1}^m \Gamma_k \ln(z - b_k) \quad (2)$$

and the complex velocity, respectively, will be

$$v = \frac{dw}{dz} = \overline{v_\infty} + \frac{1}{2\pi} \sum_{k=1}^n \frac{q_k}{z - a_k} - \frac{1}{2\pi} \sum_{k=1}^p \frac{M_k e^{\alpha_k i}}{(z - c_k)^2} - \frac{i}{2\pi} \sum_{k=1}^m \Gamma_k / (z - b_k), \quad (3)$$

where  $w$  – complex potential;  $v_\infty$  – complex velocity general circulation background (mainly zonal circulation);  $b_k$  – coordinates of vortex sources in the area of singularity;  $c_k$  – coordinates of the dipoles in the area of singularity;  $a_k$  – coordinates of the vortex points in areas of singularity;  $M_k$  – values of momenta of these dipoles;  $\alpha_k$  – orientation of the axes of the dipoles;  $\Gamma_k, q_k$  – values of circulation in the vortex sources and vortex points, respectively.

In the scheme by Orta [17] the Hadley circulation cell in angular momentum in the north part runs into a zone of the Arctic front, and at the time of the lithosphere it is included in the coverage of the polar front. Convergence of these atmospheric fronts could then close the cycle of atmospheric angular momentum balance in the same frequency range of atmospheric fluctuations without giving effect by an ocean and the lithosphere. Of course, the Hadley tropical cell carries teleconnection of the polar front with southern process by means of the link mechanism which is similar to link between the tropical and polar fronts or the Hadley tropical cell with a cell Hadley of temperate latitudes.

The balance of angular momentum in conditions of the close convergence of the Arctic and Polar fronts over the ocean (which is almost always in all seasons and over the continents in the summer and in the transition seasons) is largely respected by centrifugal "pull" moisture along the front section of the polar front to south of the center of the cyclonic-depressive these. The cited mechanism for atmospheric front previously was worked out in [14]. Total mass flux in a separate cloud as well as cloud system, is determined by the Arakawa's model.

If  $A$  is a work of the convective cloud then it consists of the convection work and work of down falling streams in the neighbourhood of cloud:

$$\frac{dA}{dt} = 0 = \frac{dA}{dt}_{conv} + \frac{dA}{dt}_{downstr}. \quad (4)$$

It is obvious that

$$\frac{dA}{dt}_{downstr} = \int_0^{\lambda_{max}} m_B(\lambda') K(\lambda, \lambda') d\lambda',$$

Here  $m_B(\lambda)$  is an air mass, drawn into a cloud with velocity of drawing  $\lambda$ ; if

$$\frac{dA}{dt}_{downstr} = F(\lambda) \int_0^{\lambda_{max}} K(\lambda, \lambda') m_B(\lambda') d\lambda' + F(\lambda) = 0 \quad (5)$$

is an mass balance equation in the convective thermion and  $K(\lambda, \lambda')$  is a nucleus of integral

equation (1), which defines dynamical interaction between neighbour clouds then:

$$\beta \int_0^{\lambda_{\max}} K(\lambda, \lambda') m_B(\lambda') d\lambda' + F(\lambda) = m_B(\lambda) \quad (6)$$

This is the Arakawa type equation with accounting for air streams superposition of synoptic process. Its solution is as follows:

$$m_B(\lambda) = F(\lambda) + \beta \int_0^{\lambda_{\max}} F(s) \Gamma(\lambda, s; \beta) ds, \quad (7)$$

here  $\Gamma(x, s; \beta)$  is an resolventa of the master integral equation:

$$\Gamma(\lambda, s; \beta) = \sum_{m=1}^{\infty} \beta^{m-1} K_m(\lambda, s); \quad (8)$$

$$K_m(x, s) = \int_0^{\lambda_{\max}} \dots \int_0^{\lambda_{\max}} K(x, t_1) K(t_1, t_2) \dots K(t_{m-1}, s) dt_1 dt_2 \dots dt_{m-1} \quad (9)$$

As usually, we present a resolventa of the integral equation as an expansion in the Loran set cycle in a complex plane  $\zeta$ ; its centre coincides with the centre of the heating spot of a city and internal cycle with its periphery; external one can be moved beyond limits of recreation zone. Then resolventa is as the Loran set (*with a* as centre of converge for the Loran set):

$$\Gamma = \sum_{n=-\infty}^{\infty} c_n (\zeta - a)^n, \quad (10)$$

$$c_n = \frac{1}{2\pi i} \oint_{|\zeta|=1} \frac{f(\zeta) d\zeta}{(\zeta - a)^{n+1}}.$$

**Advanced model of low-frequency atmospheric motions.** Here we consider an advanced spectral analogue for equation of motion for dynamics of the atmosphere in the low frequency range. As it is well known, the shape of the atmospheric circulation changes its position in space, and the intensity of manifestations varies in the period up to several days, while inside it implemented processes, lasting a few minutes, such as precipitation. Hydrodynamic equations are set to reasonably high-frequency processes in the atmosphere of the evolution of the cyclonic type of education in the period up to two days, but it is not able to well describe the low-frequency processes such as change of the circulation forms.

At the same time the macroturbulent atmosphere equations are low-frequency ones in its basis and there is a lot of experience of their decision on the basis of spectral methods [3,4,14,31]. This allows you to use them for our purposes for the mathematical modeling of the changing forms of circulation and, respectively, for the mathematical parameterization homologues circulation. [13,14]. In order to solve this task, one should involve coupling moments forecasting model, which we know from the system of the Reynolds equations with implemented average and fluctuation motion.

The technique of using Reynolds tension tensors of the second rank is well known (for example, in the form of an analytical representation). The circuit equations with accounting the Coriolis force in the analytical form can be rewritten as:

$$\begin{aligned} \frac{\partial V'^2}{\partial t} = & -\frac{i}{a} \left[ \overline{V'^2 L_1(\bar{V})} + 2\bar{V} \overline{V' L_1(V')} + \overline{V'^2 L_1(V')} \right] - \\ & -\frac{i}{a} \left[ L_2(\bar{V}) \overline{V' U'} + \bar{V} \overline{U' L_2(V')} + \overline{U' V' L_2(V')} + \overline{V' U' L_2(V')} \right] + \\ & + 4\omega i \cos \theta \overline{V'^2} + \frac{2i}{a} \overline{V' L_6(\Phi')}, \end{aligned} \quad (11a)$$

$$\begin{aligned} \frac{\partial U'^2}{\partial t} = & -\frac{i}{a} \left[ \overline{V' U' L_3(\bar{U})} + \bar{V} \overline{U' L_3(U')} + \overline{U' V' L_3(U')} + \overline{V' U' L_3(U')} \right] - \\ & -\frac{i}{a} \left[ \overline{U'^2 L_4(\bar{U})} + 2\bar{U} \overline{U' L_4(U')} + \overline{U'^2 L_4(U')} \right] - 4\omega i \cos \theta \overline{U'^2} + \frac{2i}{a} \overline{U' L_5(\Phi')}, \end{aligned} \quad (11b)$$

$$\begin{aligned} \frac{\partial \overline{V' U'}}{\partial t} = & -\frac{i}{2a} \left[ \overline{V'^2 L_3(\bar{U})} + 2\bar{V} \overline{V' L_3(U')} + \overline{V'^2 L_3(U')} \right] - \\ & -\frac{i}{2a} \left[ \overline{V' U' L_4(\bar{U})} + \bar{U} \overline{V' L_4(U')} + \overline{U' V' L_4(U')} + \overline{V' U' L_4(U')} \right] + \\ & + \frac{i}{a} \overline{V' L_6(\Phi')} - \frac{i}{2a} \left[ \overline{U'^2 L_2(\bar{V})} + 2\bar{U} \overline{U' L_2(U')} \right] - \\ & -\frac{i}{2a} \left[ \overline{U' V' L_1(\bar{V})} + \bar{U} \overline{V' L_1(V')} + \overline{V' U' L_1(V')} + \overline{U' V' L_1(V')} \right], \end{aligned} \quad (11c)$$

where

$$\begin{aligned} L_j = & \frac{\partial(\dots)}{\partial \theta} - (-1)^j \frac{i}{\sin \theta} \frac{\partial(\dots)}{\partial \lambda} + b_j \operatorname{ctg} \theta(\dots), \\ & b_j=1, j=1,4; b_j=-1, j=2,3; b_j=0, j=5,6. \end{aligned}$$

In many earlier papers (see for example, [10]) authors used the simplified approximation, which results to remaining only two operators, say, the equation (11c)

$$\frac{\partial \overline{V' U'}}{\partial t} = \frac{i}{a} \overline{V' L_6(\Phi')}, \quad (12)$$

expressing  $\Phi'$  through  $\varphi$  complex potential of the velocity  $w$ , and the velocity components  $V'$  - in terms of functions  $\psi$  of the same velocity potential. We suppose that this procedure should be replaced by more consistent one that provides an advanced level of a theory.

Naturally, the equations for tensor of the turbulent tensions:

$$\begin{aligned} \frac{\partial \overline{u'_i u'_j}}{\partial t} + \frac{\partial}{\partial x_k} \left( \overline{u'_k \cdot u'_i u'_j} + \overline{u'_k u'_i u'_j} \right) + \frac{\partial \overline{p' u'_i}}{\partial x_j} + \frac{\partial \overline{p' u'_j}}{\partial x_i} = \\ = -\overline{u'_i u'_k} \frac{\partial \overline{u'_j}}{\partial x} - \overline{u'_j u'_k} \frac{\partial \overline{u'_i}}{\partial x} + p' \left( \frac{\partial \overline{u'_i}}{\partial x} + \frac{\partial \overline{u'_j}}{\partial x} \right); \end{aligned} \quad (13)$$

The kinetical energy of fluctuations is  $b^2 = \overline{u'_k u'_k}$ . The corresponding eq.:

$$\frac{\partial b}{\partial t} + \frac{\partial u'_k b^2}{\partial x_k} + \frac{\partial}{\partial x_k} \left( \overline{u'_k u'_i u'_j} + 2\overline{u'_k p'} \right) = -2\overline{u'_k u'_i} \frac{\partial u'_i}{\partial x_k} - 2 \frac{g}{\Theta_0} \overline{w' \theta'} \quad (14)$$

Advection	Turbulent diffusion	Effect of forces of tension	Interaction of the Reynolds tension and averaged motion	Generation for account for swimming forces
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Here  $\theta$  is potential temperature. Velocity's correlates are as follows:

$$\begin{aligned} \overline{u'_i u'_j u'_k} &= -b\lambda_1 \left( \frac{\partial \overline{u_i u_j}}{\partial x_k} + \frac{\partial \overline{u_i u_k}}{\partial x_j} + \frac{\partial \overline{u_j u_k}}{\partial x_i} \right); \overline{u'_k u'_j \theta'} = -b\lambda_2 \left( \frac{\partial \overline{u'_k \theta'}}{\partial x_j} + \frac{\partial \overline{u_j \theta'}}{\partial x_k} \right); \\ \overline{u_i \theta'^2} &= -b\lambda_3 \left( \frac{\partial \overline{\theta'^2}}{\partial x_i} \right); p' \frac{\partial \theta'}{\partial x_i} = -\frac{b}{3l_1} \overline{u_i \theta'} - \frac{1}{3} \sigma_{i3} \frac{g}{\theta_0} \overline{\theta'^2}; \\ p' \left( \frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right) &= -\frac{b}{3l_1} \left( \overline{u_i u_j} - \frac{1}{3} \sigma_{ij} b^2 \right) + cb^2 \left( \frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right). \end{aligned} \quad (15)$$

Here  $c$ ,  $l_1$ ,  $\lambda_i$  are constants which define the scales of turbulent vortexes and measure of their influence on the averaged motion and atmosphere turbulence anisotropy. Components of tensor of the turbulent tensions are ( $v_{l,n}$  - spectral modes of velocity field):

$$\begin{aligned} \mathcal{E} &= \sum_{k=1}^{\infty} \sum_{s=-k}^k V_{k,s} T_{1,s}^k \left( \sum_{q=1}^{\infty} \sum_{j=-q}^q V_{q,j} T_{1,j}^q \right) = \sum_{k=1}^{\infty} \sum_{s=-k}^k \sum_{q=1}^{\infty} \sum_{j=-q}^q V_{k,s} V_{q,j} \times \\ &\times \sum_{v=|k-q|}^{k+q} \sigma_{1,1,2}^{k,q,v} \sigma_{s,j,s+j}^{k,q,v} T_{2,s+j}^v = \overline{v'_1 v'_1} = b^2, \end{aligned} \quad (16)$$

**Concluding remarks and conclusions.** The paper presents the elements of the advanced non-stationary theory of global mechanisms in atmospheric low-frequency processes, the balance of the angular momentum of the Earth, teleconnection effects and atmospheric radio waveguides implemented into new geophysical microsystem technology "GeoMath". The strict theory must take into account the connection of tropospheric radio waveguide with atmospheric moisture circulation and thus the shape of the atmospheric circulation across the state fronts (atmospheric fronts as the main storages of moisture). Atmospheric moisture cycle is associated with the typical low-frequency performance of the process as the balance of angular momentum. Last imbalance characterizes the rotation of the atmosphere together with the Earth, which may lead to the development of meridional processes with the implementation of the mass transfer of air and steam between tropical latitudes (with a linear velocity) and slowly rotating air masses of polar latitudes (in fact it is a slow process teleconnection). Dynamics and characteristics of atmospheric radio waveguide is just related to the teleconnection and, thus, the forms of circulation, with the processes of succession of these forms (which is important in the long-term prognosis). Imbalance of angular momentum can not remain without consequences in the atmosphere due to the rather large forces involved in the desired dynamics. Naturally imbalance causes the effects of the singularity, i.e. sharp reaction of the atmosphere in an attempt to eliminate it. In any case, such a serious impact on the atmosphere, in principle, can largely cause change in the form of atmospheric circulation, which allows you to quickly redress the imbalance of angular momentum organization sufficiently rapid moisture transport and air speed of rotation from north to south to its torque.

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**Моделювання балансу кутового моменту землі, параметрів атмосферних процесів та радіохвильоводів: удосконалена нестационарна теорія**

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*Викладені елементи нестационарної теорії глобальних механізмів в атмосферних низькочастотних процесах, балансу кутового моменту Землі та ефектів телеконекції, а також атмосферних радіохвильоводів, які вивчаються на основі нової мікросистемної технології "GeoMath".*

**Ключові слова:** баланс кутового моменту Землі, атмосферні моделі, телеконекція

**Моделирование баланса углового момента земли, параметров атмосферных процессов и радиоволноводов: Усовершенствованная нестационарная теория**

**а.В.Глушков, С.В.Амбросов, Ю.Я.Бунякова, В.Ф.Мансарлийский**

*Изложены элементы усовершенствованной нестационарной теории глобальных механизмов в атмосферных низкочастотных процессах, баланса углового момента Земли, эффектов телеконнекции, а также атмосферных радиоволноводов.*

**Ключевые слова:** баланс углового момента Земли, атмосферные модели, телеконнекция