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A GENERALIZED “Z–LESS” MIXING LENGTH-SCALE FOR STABLE ATMOSPHERIC BOUNDARY LAYERS

Abstract. *Recent research suggests that the evolution of the stable ABL is still poorly understood. Certain advances in theory and modeling of the stable ABL (SABL) are assessed. Inclined strongly SABL is addressed. We show that a relatively thin and strongly SABL, as recently modeled using an improved “z-less” mixing length, can be successfully treated; the result is quietly extended to other types of SABL. Finally, a generalized “z-less” mixing length is proposed.*

Keywords: *katabatic flow, Monin-Obukhov length, numerical modelling, parameterization, Prandtl number, Richardson numbers, stratified turbulence, very stable boundary layer*

1 Introduction

The atmospheric boundary layer (ABL) is an intra- and inter-layer between various underlying surfaces, on one side (e.g., inclined terrain, urban areas, sea), and the rest of the atmosphere above. Mainly depending on forcing of the ABL, one often distinguishes various types of convective ABLs on one side, and stably stratified ABLs, i.e. the SABLs, on the other side. The focus of this study is on the very (or strongly) SABL, i.e. the VSABL [1,2,5,7,8,10] where progressively smaller eddies still play important roles in the overall behavior of the layer. On the contrary, in the typical CABL only the largest eddies determine most of characteristics of the CABL’s turbulent flow and its internal evolution; most of the dynamics pertaining to the CABL can be successfully treated via various Richardson numbers.

Going back to the SABL, small eddies governing the VSABL are difficult to measure, in order to provide statistically reliable higher-order moments (fluxes, etc.); these small eddies may be generated by a multitude of physically different processes. The corresponding turbulent structures and overall behavior of the VSABL are under complex influences emerging from strong near-surface temperature inversions, possible low-level jets (LLJ), wind meandering, unsteadiness, surface fluxes, internal boundary layers and roughness changes, buoyancy waves, etc. These features strongly affect the VSABL and thus determine its turbulence properties; hence, there are also a few types of the VSABL. Almost needless to say, the VSABL is still not well understood today [4,7,8,9,10]. Its nature, i.e. basic dynamics, physics and overall evolution are often unknown. Loosely speaking, one deals in the VSABL with the vertical scales between a few tens to a few hundreds of meters, and quasi-horizontal and temporal scales of a few kilometres and a few minutes to a several hours, respectively.

Current resolutions in numerical weather prediction (NWP), air-chemistry and climate models are still insufficient to simulate, or even emulate, bulk properties of the VSABL (which is usually thin, say, less than 100 m in its depth). Thus, we cannot learn much about the VSABL from the existing state-of-the-art numerical models either. Our current knowledge about the VSABL, and its necessary inclusion into appropriate turbulence parameterizations in NWP and the related models, is still sparse and hardly adequate. For instance, the SABL as simulated in current NWP and climate models are usually much too deep [4,9]. All these points mentioned hint the aim of this paper, which is to shed some light on a type of VSABL, and then to try to extend it, at least partially, to a broader array of VSABL flow types.

A few particular questions are to be discussed here related to the VSABL and its stratified turbulence. The focus is on the so-called "z-less" mixing length-scale, which is the relevant turbulent local mixing length above the surface layer for parameterizing the related turbulent processes. The (strongly) stable layer may be extremely thin or even non-existent in a particular VSABL [2,4,5,6,7]. One of the role models for the VSABL is the one that is driven katabatically against a calm and stably stratified background atmosphere [1,2,5,6]; this will be one of our starting points in this study. Another kind of the VSABL, still poorly understood today, is e.g. that under weak-wind strongly-stable conditions [8]. The former VSABL type, i.e., shallow katabatic flow, may often be addressed via calibrated (modified) Prandtl model [4,5,6]. In a world of ever refining resolution of NWP, climate and air-chemistry models, there are progressively lesser areas with purely horizontal land surfaces; this lends additional credibility and perspective for the modified Prandtl model as it will be done here. Pragmatic improvements to be recommended here should eventually help in preventing NWP and air-chemistry models' difficulties related to frictional decoupling and/or runaway cooling [2,4,5,9]. The latter problems are typically fixed, i.e. loosely alleviated in many current numerical models, by simply allowing for an excessive vertical diffusion in the models. In this way, being over-diffusive, the models still serve many of their main purposes (e.g. apparently simulating baroclinic instability in a proper way, filling cyclones faithfully, etc.) while erroneous SABL fields might be largely fixed retrospectively, via some sort of post-processing. Of course, this is a physically incorrect way, lacking the basic knowledge about the VSABL, and it will be shortly demonstrated and surpassed here.

This study continues on a few other recent works of the author, colleagues and the collaborators [4,8,9,10]. Its raw material (not as a whole) was presented at conference in Odessa, 2008, http://www.conf.osenu.org.ua/, dedicated to the memory of L.N. Gutman, the father of theoretical mesoscale meteorology. Some of the overall material presented there has been partially published [4], some of the results, such as e.g. a generalized "z-less" mixing length-scale, that was conceived at the conference, is a new result that has not been previously published.

2 Recently improved mixing length for the SABL

In a very recent study two very different models were successfully deployed in concert in order to improve and tune a "z–less" mixing length–scale in one of the models [4]. One is MIUU mesoscale model, i.e. a 3D fully nonlinear numerical model with a reliable higher-order turbulence parameterization scheme; a detailed explanation of the model is given in e.g. [3]. Another model is a basically analytical 1D model, arguably weakly nonlinear, with a prescribed gradually varying vertical eddy diffusivity/conductivity profile, i.e. the modified Prandtl model [5,6]. The "z–less" length mentioned, defined as a local quantity, has become [4]:

$$l_{STAB} = \min\left[a\frac{(TKE)^{1/2}}{N}, b\frac{(TKE)^{1/2}}{\$} \right], \qquad (1)$$

where the symbols have their usual meaning: *TKE* is turbulent kinetic energy, N and $\$$ are buoyancy and shearing frequency (from the absolute shear: $\$ = |S|$), respectively, $a \approx 0.5$ and $b = a/2$ – all for the gradient Richardson number $0 < \mathrm{Ri} \le 1$, $\mathrm{Ri} = (N/S)^2$; otherwise, for $\mathrm{Ri} > 1$, only the 1st term in (1) is kept. If (1) is applied for all $\mathrm{Ri} > 0$, then the 1st term in (1) will be valid only for $\mathrm{Ri} \geq 4$, provided again $b = a/2$ (the validity goes in (1) as the square of a/b due to $\mathrm{Ri} = (N/\$)^2$).

Let us plausibly define the weakly stratified SABL exhibiting everywhere $0 < \mathrm{Ri} \ll \infty$ but usually $\mathrm{Ri} \leq 1$, and the VSABL determined by its (sub)regions with $\mathrm{Ri} \gg 1$. Figure 1 displays an over-diffusive SABL in a typical mesoscale numerical model (solid curves); the profiles are taken from [4], based on their Fig. 1, simulated by MIUU model. The solid curves in Fig. 1 are obtained by using only the 1st term in (1), which was one of defaults in MIUU model [3]. The dashed curves, shown on both panels in Fig. 1 for the downslope velocity U and potential temperature θ, respectively, represent the corresponding simulation with the problem alleviated; there (1) is fully deployed. The latter simulation (dashed) is a more trustful one because it also corresponds to another model, i.e. the calibrated analytic Prandtl model result [4,6]. Both models, MIUU and Prandtl, had been previously validated independently against various observations and theories. Hence, these models qualify as very useful tools independently for studying various types of SABL flows (their complexity, basic assumptions, etc.). The main input parameters and model setups are the following. A constantly sloped terrain of –2.2° is imposed under a windless stratified background atmosphere of $\Delta\theta/\Delta z = 5\ \mathrm{K/km}$ with the surface potential temperature deficit of 6.5 K, $1.5\ \mathrm{m} \leq z < 30\ \mathrm{m}$ in the lowest $500\ \mathrm{m}$ of the atmosphere. The others, less crucial input parameters, such as the relatively small roughness length, etc. are not listed here for brevity. These were used throughout the study unless stated otherwise explicitly; the other details are in [3] or [4].

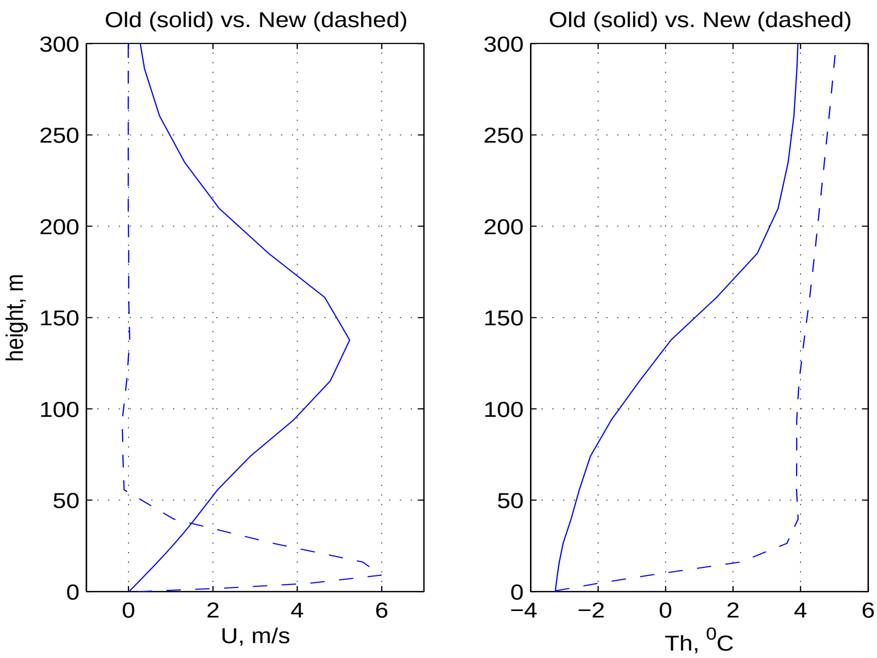

Fig. 1 – Two simulations of the same pure katabatic flow using two different parameterizations for the "z-less" mixing length in MIUU model [3,4]. The profiles of the downslope wind component U (left) and potential temperature θ (right), are shown averaged over 24 h. Over-diffusive SABL (solid) consists of an elevated LLJ and a capping inversion spread over the lowest 150 – 200 m. Using a recently proposed "z-less" mixing length (1), with *TKE* and wind shear, the SABL becomes much thinner (dashed) as expected from the theory of Prandtl.

While the over-diffusive SABL modeled, Figure 1 (solid) is much too deep, its properly modeled behavior, i.e. the VSABL (dashed), is in agreement with the calibrated Prandtl [4,6]. From a technical point of view, it is also numerically and physically stable (e.g. it does not show a sign of frictional decoupling it can reproduce diurnal cycle, etc.). It is expected that (1) ought to improve simulations for other types of SABL flows too, simply because the overall turbulence scheme deployed, a higher-order one, so called level 2.5 [2,3,4,9,10], is slope insensitive. In other words, this scheme a priori does not care whether a particular flow is katabatic (corresponding to various inclining underlying surfaces) or not. To add a point of conclusion, since the wind and its shear are, in overall, more variable than buoyancy frequency in the atmosphere, it makes much sense to deploy (1) fully, instead of only its 1st term relating the mixing length to Ozmidov length only. Once again, the basic advantage of using (1), see Fig. 1, is the prevention (dashed) of the over-diffusion of the SABL in time and height.

Next, we expand the formulation given in (1) by deriving a new generalized local mixing length-scale, then we compare it to the existing suitable mixing length-scales. In this way, we extend our most recent work about modeling of the SABL [4].

3 Generalized "z–less" mixing length

Before we proceed with further analyzing the mesoscale model simulations, we first introduce a new generalized "z-less" mixing length-scale for the SABL (and VSABL in particular) flows. This new mixing length will be derived from a simplified *TKE* equation, i.e. it will be not heuristically obtained from e.g. scaling or dimensional arguments. This proposal is a generalization of (1). Begin with the prognostic equation for *TKE* under the usual simplifying conditions: horizontal homogeneity, Boussinesq and the absence of mean vertical motions, i.e.:

$$\frac{\partial(TKE)}{\partial t} = -\overline{u'w'}\frac{\partial \bar{u}}{\partial z} + \frac{g}{\theta}\overline{w'\theta'} - \frac{\partial}{\partial z}\left[\overline{w'(\frac{p'}{\rho_0} + TKE)}\right] - \varepsilon\,, \qquad (2)$$

where the terms have their very typical meaning. Namely, the local rate of change of *TKE* on the LHS of (2) is balanced by the shear production, buoyant destruction, transport (redistribution-like) due to pressure- and turbulence-correlations and viscous dissipation, respectively. Assuming flow steadiness, and neglecting transport terms in the squared brackets, we parameterize the momentum and heat fluxes in (2) as $K_m \$$ and $K_h \$$, where K_m and K_h are eddy diffusivity and conductivity, respectively. Finally, the last term in (2) is parameterized as $\varepsilon = b\left(TKE\right)^{3/2}/\Lambda$, where b is an empirical constant and Λ is a new mixing length-scale replacing l_{STAB} from (1). Under these simplifications (2) yields:

$$0 = K_m \$^2 - K_h N^2 - \frac{b}{\Lambda}(TKE)^{3/2}\,, \qquad (3)$$

where the buoyant destruction and viscous dissipation, i.e. last two terms in (3), compete in spending *TKE* after its mechanical/shear production.

A simple 1st order closure assumes, from the absolute shear $\$$, that $K_m = a_1\Lambda^2 \$$ and $K_h = a_1\Lambda^2 \$/\mathrm{Pr}$; a_1 is a model constant and Pr is turbulent Prandtl number; typically $\mathrm{Pr} \geq 1$

in the SABL [5,6,10]. An advanced and probably better parameterization, i.e. a higher-order closure, may take a form as $K_m = a_2\Lambda\left(TKE\right)^{1/2}$ and $K_h = a_2\Lambda\left(TKE\right)^{1/2}/\mathrm{Pr}$. When plugged in (3), the corresponding $\Lambda_{1,2}$ becomes:

$$\Lambda_{1,2} = c_{1,2}\frac{(TKE)^{1/2}}{\$(1-Ri/\mathrm{Pr})^{1/(3,2)}}, \tag{4}$$

with $c_{1,2}$ being appropriate coefficients obtained from b, a_1 or a_2, respectively ($c_1 = \left(b/a_1\right)^{1/3}, c_2 = \left(b/a_2\right)^{1/2}$), the root exponent in the denominator in (4) is either 1/3 or 1/2, for the 1st or the higher–order closure, respectively. Note that *TKE* is typically forecasted in higher-order closures; meanwhile, in 1st order schemes it may be only diagnosed. Now including a very important recent finding about the SABL

$$\mathrm{Pr} \approx 0.8 + 5\mathrm{Ri}, \tag{5}$$

from [10] into (4), its denominator is justifiably expanded into binomial series because for the SABL (5) yields $\max\left(\mathrm{Ri}/\mathrm{Pr}\right) \le 0.2$. Thus, the newly proposed "z–less" mixing length–scale is approximately

$$\Lambda_{1,2} \approx c_{1,2}\frac{\left(TKE\right)^{1/2}}{\$}\left(1+\frac{\mathrm{Ri}}{(3,2)\,\mathrm{Pr}}\right), \tag{6}$$

which is a modification of (1); again, the indices correspond straightforwardly to those in (4), i.e. $\Lambda_{1,2}$ to $c_{1,2}$ relating to the 2nd term in the brackets to $\mathrm{Ri}/\left(3\,\mathrm{Pr}\right)$ or $\mathrm{Ri}/\left(2\,\mathrm{Pr}\right)$, respectively. Note that there is a whole class of the alike parameterizations, i.e. between 1st and 2nd order closures, allowing for the same basic formulation (6), namely, $\Lambda \sim \left(TKE\right)^{1/2}/\$$.

For K_m and K_h parameterized in (3) as e.g. $K_m = a_3\left(TKE/N\right)$ and $K_m = a_3\,TKE/\left(\mathrm{Pr}\,N\right)$, which also makes much sense for the VSABL, instead of (4), we end up with

$$\Lambda_3 = c_3\frac{(TKE)^{1/2}}{\$}\frac{\mathrm{Ri}^{1/2}}{(1-\mathrm{Ri}/\mathrm{Pr})}, \tag{7}$$

where c_3 is obtained in the same manner as $c_{1,2}$ ($c_3 = b/a_3$). Since (5) allows a binomial series of the denominator, like in (6) and again based on the smallness of the ratio Ri/Pr as in (5), one may also expand (7). We conclude that most of meaningful parameterizations between 1st and 2nd order turbulence closure schemes for the VSABL are well treated with a "z–less" mixing length scale of the type:

$$\Lambda = const\frac{(TKE)^{1/2}}{\$}f(\mathrm{Ri},\mathrm{Pr}), \tag{8}$$

with $0 < const < 1$ and $f(\mathrm{Ri}, \mathrm{Pr})$ as a simple function, or even a simpler series expansion, already given for two classes as $\approx 1 + \mathrm{Ri}/(3\,\mathrm{Pr})$, or $\approx 1 + \mathrm{Ri}/(2\,\mathrm{Pr})$; in the third case discussed this $f \approx (\mathrm{Ri})^{1/2}(1 + \mathrm{Ri}/\mathrm{Pr})$. For both 1st order– and higher–order closure schemes, the respective single coefficient on the RHS of (8) is a priori known number from the respective definitions of eddy diffusivities in each particular NWP or climate model deployed.

Mesoscale models with advanced higher-order turbulence closure schemes, as e.g. MIUU model [3,9], typically have a multiple choices for obtaining eddy diffusivity and conductivity under stable conditions; meanwhile, a suitable set of options and entering coefficients is already accommodated implicitly with the proposed Λ. Nonetheless, any combination of the parameterizations discussed end up with (8), i.e. $\Lambda \sim (TKE)^{1/2}/\$$. This is provided by the systematic reduction of TKE, (2) to (3), which yields the balance of three terms deployed for Λ. A test with MIUU model shows that Λ_2 from (6) behaves in accordance with the expectation, i.e. there is no distinguishable difference between the katabatic flow simulation already displayed using (1), and the one with (6), Fig. 2. It cannot be overstressed that the katabatic flow fields from MIUU model displayed in Fig. 2

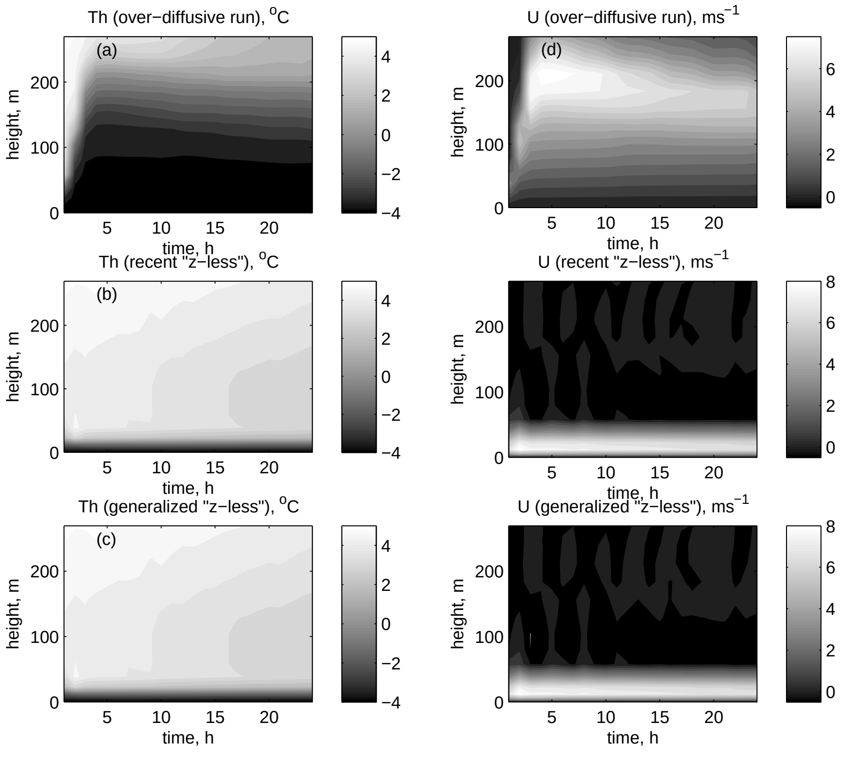

Fig. 2 – Left (a), (b), (c): potential temperature and right (d), (e), (f): downslope wind component, U, vs. time and height simulated using MIUU model. Details from Fig. l, solid, are in the upper two panels (old, over-diffusive results); dashed, are in the middle two panels (recent, correct). In Fig. 1 these results were averaged in time. Lowest two panels (c), (f) are obtained using Λ_2 from (6); these results are almost undistinguishable from those in the middle, (b), (d), giving approval to the derivation of the generalized "z–less" mixing length Λ.

correspond very well to the calibrated Prandtl model [5]. Of course, all the fields modeled are coupled among themselves in the dynamically consistent way through the governing equations. The input parameter set assigned to MIUU model is: the Coriolis parameter, slope angle, surface potential temperature deficit and background temperature gradient $(f, \alpha, C, \Delta\theta/\Delta z) = \left(10^{-4}\,\mathrm{s}^{-1}, -2.2^{\circ}, -6.5^{\circ}\mathrm{C}, 5\cdot 10^{-3}\,\mathrm{K}(\mathrm{km})^{-1}\right)$.

Top four panels in Fig. 2 show time–dependent details of the modeled flow; the time averaged fields were previously displayed in Fig. 1 for the motivation purpose. The lowest two panels, using the new generalized "z-less" mixing length from (6), correspond nicely to the recent result [4], two middle panels, thus giving the credentials to this study. The main lines of the corresponding discussion have been already presented. Next, we display a few additional flow fields from the same model run and organize them in the same fashion as in Fig. 2. Figure 3 shows across-slope wind component, V, left column (Fig. 3a to 3c), absolute air temperature, T, middle column, and the mixing length, Λ, right column. The upper panels correspond again to the old, over-diffusive run, the middle-row panels relate to our recent results [4] and the lowest panel pertains to the new, generalized result of this study.

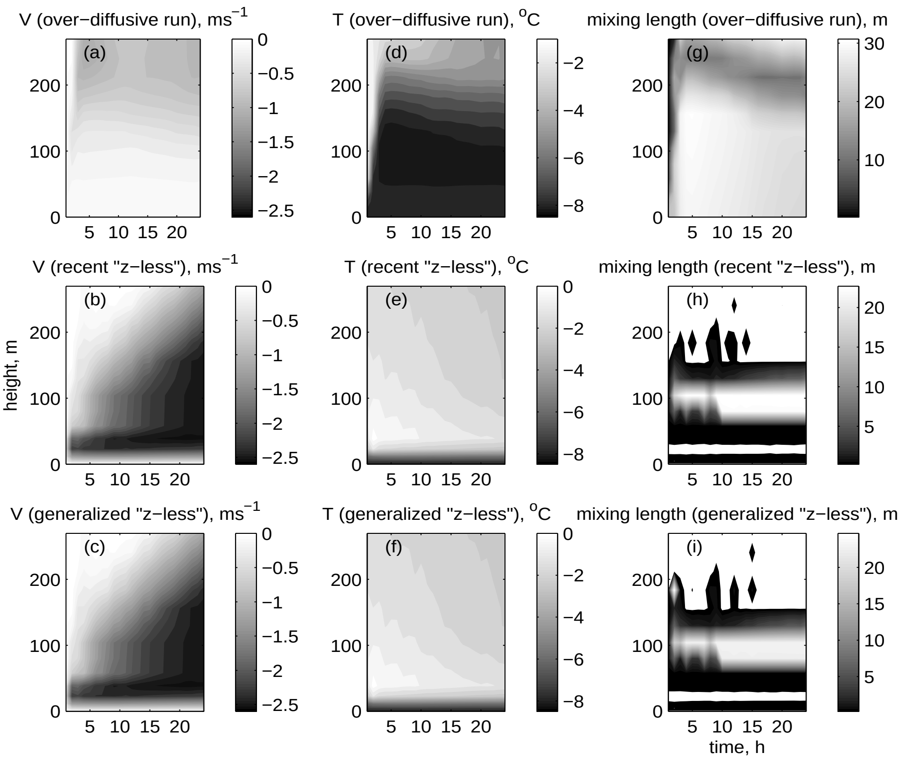

Fig. 3 – Same as Fig. 2 but now for the across-slope wind component V, left column, absolute air temperature, T, middle column, and the mixing length, Λ, right column. The middle row (b), (e), (h) is obtained with (1), as in [4]; the lowest three panels (c), (f), (i) are obtained using (6).

Simple katabatic flows, e.g. as those displayed here (Boussinesq, hydrostatic, quasi-1D, without large-scale pressure gradient, all for constant: surface potential temperature deficit, slope, and roughness), if sufficiently persistent, e.g. over long glaciers during the polar night, might generate permanent effects on the troposphere [6]. During persistent katabatic forcing, the across-slope wind component V is induced due to the Coriolis effect; V diffuses upwards without a well-defined spatio-temporal scale, Fig. 3b, 3c. This could affect, in principle, the whole troposphere, all the way upward to the polar vortex (after ~ 180 days of polar night). Note that this gradual upward diffusion of the *V*-component is absent in the otherwise over-diffusive SABL, Fig. 3a, 3d and 3g. Although the temperature gradually decreases through the lowest few hundreds of meters of the adequately modeled VSABL, Fig. 3e and 3f, it never over-diffuses upward, as in Fig. 3d, but it remains strongly stratified through the lowest few tens of meters where the katabatic LLJ exists, Fig. 2e and 2f. This is accentuated by the significantly smaller corresponding mixing length scale using (1) and (6), Fig. 3h and 3i, respectively. At the nose of LLJ, i.e. at only 15 to 20 m above the surface, Fig. 2e and 2f, the mixing length is < 1 m, typically $\Lambda \sim 0.2\ \mathrm{m}$, Fig. 3h and 3i. The lower four panels in Fig. 2 and the lower six panels in Fig. 3 show that much stronger gradients occur within this VSABL modeled with (1) or (6), than with the old mixing length formulation using only the 1st term in (1), the uppermost panels in Fig. 2 and 3. Note that stronger gradients, sharper LLJ and shallower near-surface inversions are the usual characteristics of the katabatic type of VSABL flows [1,2,4,5,6]. There it is the LLJ and its shear that govern the turbulence properties, not e.g. a distance from the surface. Describing the VSABL with e.g. Blackadar type of the mixing length-scale will never be successful because of allowing for too much vertical mixing. Even a more sophisticated local length-scale, e.g. “z–less” length based or related to Ozmidov scale, like the 1st term of (1), will also often be wrong because of excluding the most relevant scale, i.e. the wind shear explicit effect.

A few remarks and side notes follow before the final conclusion. An enhanced Λ sensitivity to shear effects, which generate but also limit the turbulent eddies, (6) through (8), can be beneficial in sensing other, even non-local features of turbulence, such as transport and redistribution. While (6) through (8) might have difficulties in treating turbulent mixing for wind shear diminishing faster than $(TKE)^{1/2}$, occurring in some strongly-stratified weak-wind conditions, it remains to be checked if the newly proposed generalized “z–less” length-scale will lead some practical betterments in modeling VSABL. It seems that the latter type of VSABL is governed by mostly unknown physics [7,8]. Without suitable measurements there, yielding reliable statistics, we do not even know if the relatively weak turbulence in the weak-wind VSABL is transported or redistributed from elsewhere and then only partially destroyed in this VSABL. Other scenarios seem plausible too, vaguely relating to e.g. flow meandering, internal boundary layers, buoyancy waves (re)generation and modification or even alteration, etc. Almost needless to say, we must first understand these processes in order to model them properly, or at least to parameterize them adequately in our current mesoscale and climate models. These (mostly unknown) transports could be related to buoyancy-(infra-)sound waves, purely stochastic processes, anomalous (fractional) diffusion, etc.

3 Conclusion

A few aspects of the SABL are discussed in this work, the focus being on the numerical modeling and parameterization of turbulence in the SABL. The “classical” SABL is weakly stratified, i.e. $\mathrm{Ri} \ll \infty$, usually, $0 < \mathrm{Ri} \leq 1$, and it is typically modeled well nowadays [10]. However, strongly stable cases of the SABL, i.e. the VSABL, where typically $\mathrm{Ri} \gg 1$, is generally not understood well [4,5,7,8,9,10]. Over-diffusive and too deep SABL flows in models are addressed; a newly proposed local, so called, generalized “z–less” mixing length scale apparently remedies a large part of the over-diffusion problems. A thin and relatively sharp VSABL flow regime is obtained using the new length–scale (8), giving almost the same result as a recently recommended (1), see [4]. The particular VSABL type modeled here is katabatic flow consisting of the LLJ imbedded into the near-surface inversion. Since the turbulence parameterization scheme deployed [3,4] is slope insensitive, the betterment offered here is of a general nature (i.e. not only pertaining to katabatic flows); other tests are necessary, however. However, the improvement offered here may be extended to other types of SABL flows.

The newly proposed mixing length-scale, (6) through (8), explicitly includes the vertical shear of horizontal wind. It is basically given as $\Lambda \sim (TKE)^{1/2} / \$$, derived from a few most recent works [4,9,10] indicating a few obvious shortcomings of the current turbulence parameterizations for the SABL and its turbulence effects as modeled in NWP, air-chemistry and climate models. This generalized "z–less" mixing length-scale, compatible with the recently offered length-scale (1), remains to be checked against measurements through suitable numerical simulations and various tests. Tentative simulations for pure katabatic flows using the newly proposed Λ_2 from (6) alternating with Λ_3 from (7) display promising results concurring with (1) in agreement with [4]. Hopefully, (6) or (8) would be soon tested and implemented in the current NWP and air-chemistry models, such as WRF, EMEP, etc.

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Обобщенный путь смешения, не зависящий от координаты z, для устойчивого пограничного слоя атмосферы

Аннотация. *В последних исследованиях утверждается, что эволюция устойчивого пограничного слоя атмосферы (ПСА) все еще недостаточно изучена. В данной статье дается оценка некоторым достижениям в теории и моделировании устойчивого ПСА. Рассматривается также устойчивый ПСА над наклонной подстилающей поверхностью. В настоящем исследовании показано, что при использовании уточненного пути смешения, не зависящего от координаты z, модель хорошо воспроизводит относительно тонкий и сильно устойчивый ПСА, а полученные результаты могут быть успешно распространены на другие типы устойчивого ПСА. В работе также предлагается обобщенный путь смешения, не зависящий от координаты z.*

Ключевые слова: *катабатическое течение, длина Монина–Обухова, численное моделирование, параметризация, число Прандтля, числа Ричардсона, стратифицированная турбулентность, сильно устойчивый ПСА.*